

distance of ten to a hundred meters was provided by the engine device of the primary satellite.

Intercosmos-25 successfully obtained many interesting results. All scientific units operated normally. A series of active experiments on studying emission from plasma and electron beams and their detection was carried out by the subsatellite.

In the 1990s, after the dissolution of the Soviet Union, the organizational structure of the Intercosmos Council was de jure dissolved. The Council for Mutual Economic Assistance, the Warsaw Pact ceased to exist. In most of the countries participating in the Intercosmos program, the economical and political regimes changed, but scientific and personal relations between scientists have persisted.

One of merits of the Intercosmos program was the increasing from year to year internationalization of Soviet cosmonautics. For example, in the Vega (Venus-Galley) project realized in 1984, in addition to the countries permanently collaborating on the Intercosmos program, scientists from Austria, Germany, and France participated in the manufacturing of the scientific payload installed on automatic interplanetary stations Vega-1 and Vega-2.

The first part of the flight program of these stations was aimed at exploring the atmosphere and surface of Venus. To this effect, balloon-borne probes were used for the first time. During the second part of the program, the stations approached the Galley comet and after 450 days of flight, in March 1986, they passed near the comet's core at a distance of about 10,000 km. In the experiments, the size and form of the cometary core were determined along with the surface properties and the temperature and chemical composition of the gas, dust, and other parameters of the comet. In addition, television pictures of the comet were recorded and transmitted to Earth.

Planetary projects Phobos and Mars-96 and astrophysical missions Kvant and Spektr series were prepared under a broader international cooperation.

The Intercosmos program was in fact continued in the middle of the 1990s during the realization of the largest international project, Interbol (Fig. 11), in which 14 countries participated. The project became a part of a broad international program coordinated by the Inter-Agency Consultative Group (IACG), including representatives of the ESA, NASA, the Russian Space Agency, and the Institute of Space and Aeronautical Science (Japan).

The Interbol multisatellite project became one of the most successful missions aimed at the studies of physical processes in near-Earth space during the whole history of solar-terrestrial relations research in the Soviet Union and Russia. During the realization of this project, a system consisting of two pairs of satellites was constructed and realized: the primary satellite, Interbol-1, with subsatellite Magion-4, and the subsidiary satellite Interbol-2 with subsatellite Magion-5. This setup allowed making simultaneous measurements in different parts of the Earth's magnetosphere and enabled the separation of spatial and time variations of the measured parameters.

The Interbol project collected the unique (in significance, volume, and quality) experimental material. It became possible first of all due to much more extensive data transmission capacity from the spacecraft to the ground compared to the previous Prognoz series, and to simultaneous multisatellite observations from both close and remote distances in the Earth's magnetosphere. The lifetime of the

satellites was much longer than their assured life. These factors were crucial for providing a high scientific level of the results obtained. The results of the conducted research were published in more than 500 papers diverse in themes and approaches to the analysis of measurement results.

During the realization of this project, new important data had also been obtained on the long-term impact of different space factors on the onboard payload and functionality of technical systems, which yielded valuable recommendations for developers of space technologies.

Presently, several new large international space projects are under preparation. The impetus given by the Intercosmos program and personally by V A Kotel'nikov was crucial for surviving the hard times of the 1990s and, despite the political woes, for preserving and continuing scientific collaboration with colleagues from Eastern and Western Europe at a new and higher level. Now full scientific collaboration in space has been restored with Poland, Bulgaria, and France. A new agreement with the Czech Republic is under preparation.

The experience of Intercosmos turned out to be also very important in establishing relations with CIS countries.

The authors acknowledge Yu I Zaitsev and V S Kornilenko for the help in the preparation of the text of the report for publication.

PACS numbers: **01.65.+g**, **02.70.-c**, **89.70.-a**  
DOI: 10.3367/UFNe.0179.200902j.0216

## Development of Kotel'nikov's sampling theorem

N A Kuznetsov, I N Sinitsyn

### 1. Introduction

The name of Academician V A Kotel'nikov means a full epoch in the development of communication systems, radio engineering, and radiophysics. His greatest research achievements had a considerable impact on scientific progress throughout the world. Among them, one should mention his *sampling theorem* [1], the theory of potential noise immunity, which provided scientists and engineers with an instrument for the synthesis of optimal systems for signal processing in communication systems, radar, radio navigation, and other fields, and finally, the development of planetary radars admitting of basic astronomic research with their help.

In 1932, Kotel'nikov prepared a conference report, "On the transmission capacity of 'ether' and wire in electric communications." In this report, he gave the first formulation of the famous sampling theorem, one of the basic theorems in communication theory. This report was published, as a small edition, in 1933.

Let us consider below recent developments of the sampling theorem, its relation to the filtering of continuous signals using discrete observations, and the informational aspects of numerical simulation in the digital processing of complex signals.

### 2. Kotel'nikov's sampling theorem

**Sampling theorem in the time domain.** A continuous signal  $x(t)$  whose spectrum is limited by a maximal frequency  $F_m$  can be unambiguously and losslessly restored from its discrete samplings taken with a rate of  $F_{discr} \geq F_m$ . The algorithm of

interpolating this signal from discrete samplings spaced at  $\Delta t_m$  time intervals is given by

$$x(t) = \sum_{k=-\infty}^{\infty} x(k\Delta t_m) \frac{\sin[\omega_m(t - k\Delta t_m)]}{\omega_m(t - k\Delta t_m)}, \quad (1)$$

where  $\omega_m = 2\pi F_m$  is the Kotel'nikov frequency. The sampling interval  $\Delta t_m = 1/(2F_m)$  is often termed the Kotel'nikov interval.

**Sampling theorem in the frequency domain.** For a signal  $x(t)$  limited by time  $|t| < T$ , its continuous spectrum  $s_x(f)$ , is represented as

$$s_x(f) = \sum_{k=-\infty}^{\infty} s_x(2\pi k\Delta f) \frac{\sin 2\pi T(f - k\Delta f)}{2\pi T(f - k\Delta f)}, \quad (2)$$

where  $\Delta f$  is the frequency sampling interval.

Independently, the sampling theorem was discovered in 1949 by the outstanding American scientist C Shannon, who was the founder of the information theory, an important part of the communication theory. This theorem was extremely valuable for communication technology. It is worth noting that as a special mathematical result of the function interpolation theory, this theorem had been formulated as early as the beginning of the 20th century by British mathematicians E T Whittaker and J M Whittaker. However, this great scientific achievement is rightly attributed to the names of Kotel'nikov and Shannon, since it is only their discovery of the sampling theorem that enabled engineers to develop digital systems, which later in the 20th century made a revolution in electric communications and digital signal processing.

### 3. Applications of Kotel'nikov's theorem

Kotel'nikov formulated the sampling theorem trying to answer the following principal question: What minimal bandwidth is required for transmitting through a communication channel a signal whose spectral band is strictly limited? Today, this theorem is generally recognized as one of the fundamental results of digital signal processing (DSP) in the communication theory.

The theorem has an extremely broad field of applications. As an example, let us mention discrete communication channels and devices for digital information writing, aimed at the transmission and recording of acoustic signals. According to Kotel'nikov's theorem, the following sampling rates have been chosen:

- 8000 Hz for the telephone;
- 22,050 Hz for radio;
- 44,100 Hz for the audio CD.

Broad application of these devices is evident from the fact that in 2003, according to the Recording Industry Association of America (RIAA), 749.9 million CDs were sold.

### 4. Generalizations of Kotel'nikov's theorem

The design and exploitation of new digital devices for recording, transmitting, and reproducing continuous signals stimulated researchers to develop new DSP algorithms, such as evaluation and simulation. First of all, it should be noted that the Kotel'nikov theorem only solves the problem of interpolating a function in the course of sampling it in an infinite time interval,  $-\infty < t < +\infty$ . In practice, one always deals with a *finite observation time interval*, and it is necessary to not only interpolate a function on a finite interval but also

perform filtration, i.e., evaluate the function at some instants of time from its samplings in the interval from 0 to  $t$ , as well as extrapolation, i.e., predicting the function values at instants of time  $T > t$  from its samplings in the interval from 0 to  $t$ . Therefore, it was important to develop algorithms for restoring the values of a function within the intervals between discrete samplings, i.e., at the instants  $t$  of time falling between the values of  $t_i$  and  $t_{i+1}$ . In other words, the infinite series (1) was to be substituted by a finite series. For practical implementations, extrapolators of various complexity were developed, most of them in the form of power series, Lagrange polynomials, splines, atomic functions, etc. [2].

Results obtained by Kotel'nikov stimulated a series of works aimed at eliminating the following restrictions adopted in the proof of Kotel'nikov's theorem [3–7]:

- (1) fixed zero initial sampling;
- (2) infinite spectrum of real stochastic signals;
- (3) complexity of calculations for restoring the function by means of series (1) and (2);
- (4) nonuniformity of the samplings;
- (5) bunching of the samplings;
- (6) impossibility to find the statistical characteristics of errors during the sampling;
- (7) impossibility to take into account the errors of measuring the function at sampling points  $t_i$ ;
- (8) impossibility to take into account the errors caused by a limited digitization capacity for the series (1) and (2), etc.

### 5. Filtration and simulation of continuous processes from discrete observations and Kotel'nikov's theorem

It is well known [8] that principally new prospects for creating algorithms for filtering a continuous signal from discrete measurements opened up after the works by R Kalman, where the desired signal was represented as a solution to a linear stochastic differential equation. Let us consider a system whose state at any instant of time is unambiguously determined by a certain set of phase variables (output coordinates and their derivatives), which are not measurable directly. In addition, there are some variables related somehow to the state of the system, which can be measured at some discrete instants of time with a given accuracy. Kalman considered coordinate filtration in the conditions where the desired signal was described by continuous stochastic differential equations and the measurements were continuous, as well as in cases where the desired signal was described by a recurrent random sequence (the discrete-variable analogue of a stochastic differential equation) and measurements occurred at discrete instants of time. In Ref. [9], the problem of controlling such measurements was formulated and solved.

Computer realizations of a Kalman filter rely on the following two important facts.

(1) The matrix amplification coefficient of a filter is found by solving the discrete nonlinear Riccati equation. The conditional covariance matrix for the filtration errors is 'desymmetrized' due to the finite capacity of digital computers (DCs).

(2) In a computer simulation of a real process, models with continuous sets of states are substituted by models with discrete sets of states, which causes additional distortions of the results. The reason of such a complication is that any discretization procedure represents a fundamentally nonlinear (moreover, discontinuous) mapping, which introduces considerable distortions into the processed signal. *The effect of such distortions can also be qualitatively explained in terms*

of Kotel'nikov's theorem; however, a quantitative description of this effect is connected with considerable technical difficulties and fundamental theoretical problems. It should be noted that, unfortunately, this phenomenon is not sufficiently considered in the literature; therefore, let us dwell on it a bit more.

## 6. Two fundamental questions in computer simulations

**First.** The final goal of computer simulation is obtaining information about the object under study. But then, one should take into account that while it is often possible to pass from one continuous model to another without the loss of information (homeomorphous changes of variables, etc.), passing from a continuous object to a discrete model, as a rule, leads to information loss. A simple example is discretization of a reversible linear system on a uniform lattice: as a rule, it is an irreversible mapping. Another example considers the main information characteristic of a dynamical system, its entropy. The entropy is a measure of the exponential increase in the ratio of the number of different trajectories of the system to their length. However, any unambiguous spatial discretization of a system allows only a limited number of infinite trajectories, and the definition of entropy becomes meaningless in this case. Here, we have an evident contradiction between a continuous object and its discrete model; also evident is the necessity to improve the methods of evaluating the entropy of a continuous system from its discretizations. Notice that although different methods of solving this problem have already been proposed, the general task is very difficult to solve. In other cases, the conflict may be less evident but not less dangerous. Hence, the first fundamental question of every computer simulation is: *What is the information loss for the chosen scheme of passing from a continuous object to a discrete one?*

**Second.** The question is related to the continuous-mathematics analogues of robustness and structural stability. In continuous simulations, if one omits verbal descriptions, this is the question of whether one property or another of the object is tolerant to continuous, smooth, etc. (but necessarily small in some continuous sense) perturbations. However, if we accept that the main point of computer simulations is related to information, we should also pose the following question: *Can we guarantee the information robustness of the chosen scheme of passage from the continuous object to the discrete one?*

Probably, thorough analysis of these questions will be one of the strategic areas of natural sciences in the nearest decades. To give an overall description of the situation in this field is a hopeless task, even more so to predict its development. Some initial progress in this area is reported in Refs [10–12].

This work was supported in part by RFBR (projects Nos 06-01-00256 and 07-07-00031).

## References

1. Kotel'nikov V A "O propusknoi sposobnosti 'efira' i provoloki v elektrosvyazi" ("On the transmission capacity of 'ether' and wire in electric communications"), in *Vsesoyuznyi Energeticheskii Komitet. Materialy k I Vsesoyuznomu S'ezdu po Voprosam Tekhnicheskoi Rekonstruktsii Dela Svyazi i Razvitiya Slabotochnoi Promyshlennosti* (The All-Union Energy Committee. Materials for the 1st All-Union Congress on the Technical Reconstruction of Communication Facilities and Progress in the Low-Currents Industry) (Moscow: Upravlenie Svyazi RKKA, 1933) p. 1; second edition: *O Propusknoi*

- Sposobnosti 'Efira' i Provoloki v Elektrosvyazi* (On the Transmission Capacity of 'Ether' and Wire in Electric Communications) (Moscow: Inst. Radiotekhniki i Elektroniki MEI (TU), 2003); *Usp. Fiz. Nauk* **176** 762 (2006) [*Phys. Usp.* **49** 736 (2006)]
2. Kravchenko V F, Rvachev V L *Algebra Logiki, Atomarnye Funktsii i Veivlety v Fizicheskikh Prilozheniyakh* (Algebra of Logics, Atomic Functions and Wavelets in Physical Applications) (Moscow: Fizmatlit, 2006)
3. Whittaker E T "On the functions which are represented by the expansion of interpolating theory" *Proc. R. Edinburgh* (35) 181 (1915)
4. Balakrishnan A V "A note of the sampling principle for continuous signals" *IEEE Trans. Inform. Theory* **3** 143 (1957)
5. Belyaev Yu K "Analiticheskie sluchainye protsessy" ("Analytical random processes") *Teoriya Veroyatnostei Ee Primeneniya* (4) 402 (1959)
6. Beutler F J "Sampling theorems and basis in a Hilbert space" *Inform. Control* (4) 97 (1961)
7. Jerri A J "The Shannon sampling theorem — its various extensions and applications: a tutorial review" *Proc. IEEE* (65) 1565 (1977)
8. Sinityn I N *Fil'try Kalmana i Pugacheva* (Kalman and Pugachev Filters) (Moscow: Logos, 1st ed. — 2005, 2nd ed. — 2007)
9. Grigor'ev F N, Kuznetsov N A, Serebrovskii A P *Upravlenie Nablyudenyami v Avtomaticheskikh Sistemakh* (Control of Observations in Automatic Systems) (Moscow: Nauka, 1986)
10. Kozyakin V S, Kuznetsov N A "Dostovernost' komp'yuternogo modelirovaniya s tochki zreniya teorii informatsii" ("Reliability of computer simulations from the standpoint of information theory") *Informatsionnye Protsestry* **7** 323 (2007); <http://www.jip.ru/2007/323-368-2007.pdf>
11. Vladimirov I "Quantized linear systems on integer lattices: Frequency-based approach. Part I", CADSEM Report 96-032 (Geelong, Australia: Deakin Univ., 1996)
12. Diamond P, Vladimirov I "Higher-order terms in asymptotic expansion for information loss in quantized random processes" *Circuits, Systems, Signal Process.* **20** 677 (2001)

PACS numbers: **43.30.** + **m**, **43.58.** + **z**, 91.50.Ga  
DOI: 10.3367/UFNe.0179.200902k.0218

## Remote sensing of sea bottom by hydroacoustic systems with complex signals

V I Kaevitser, V M Razmanov

### 1. Introduction

This report deals with various aspects of applying complex sounding signals with linear frequency modulation (LFM) in hydroacoustic systems (including a multielement antenna) for the exploration of the ocean floor. The report presents a review of theoretical and practical results obtained by authors recently in the course of the development, testing, and implementation under different conditions of the following hydroacoustic systems: acoustic low-frequency linear profilographs, surveillance and interferometric side-looking sonars (SLSs), and multibeam echo sounders.

The radar exploration of planets that was conducted starting in the late 1950s by the group of scientists under the leadership of V A Kotel'nikov resulted in establishing at the Institute of Radioengineering and Electronics (IRE) of the USSR Academy of Sciences (now the Russian Academy of Sciences — RAS) a new field of research — remote mapping of extended objects by high-energy complex sounding signals and digital methods of coherent processing of echo signals. The digital methods of signal synthesizing, recording, and processing, used earlier for planetary radar, in late 1970s were