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# V A Kotel'nikov and his role in the development of radiophysics and radio engineering

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## 1. Introduction

It is not an easy task to write about V A Kotel'nikov's role in the development of radiophysics and radio engineering. This difficulty stems both from the fact that he was involved in the making and development of a diversity of areas and from the scientific results, part of which were obtained more than 70 years ago and which appear 'obvious' from the modern point of view. The difficulty is also related to the fact that Kotel'nikov was not a 'publication-lover'. In particular, his famous theorem was never published properly at all, and his classic work on potential noise immunity was not published until 1956, ten years after its completion.

## 2. Theorem

In 1932–1933, the 25-year-old engineer Kotel'nikov conceived the idea of whether it is possible to transmit without distortions a signal in a frequency band which is narrower than is allowed by transmission 'on one sideband'. In the modern view, this signifies the possibility for distortion-free transmission of signals through a channel whose spectral transmission capacity is narrower than the spectral width of the signal. To us, this sounds absurd, but at that time (1933), when the problems of spectral filtration were not quite clear to engineers, such a formulation of the problem appeared reasonable. In this connection, mention should be made of the debate at that time about whether an amplitude-modulated signal is a sinusoidal oscillation with a slowly varying amplitude or a set of spectral components. The findings of Kotel'nikov's investigation were formulated in the form of a report "On the transmission capacity of 'ether' and wire in electric communications" prepared for the 1st All-Union Congress on the Technical Reconstruction of Communication Facilities and Progress in the Low-Currents Industry. The Congress was never held, but the materials submitted to the Organizing Committee were published [1], which served as official confirmation of Kotel'nikov's priority of proving the famous sampling theorem.

The work in fact contained seven theorems, but all of them were to an extent the development of the principal theorem which states that any function $f(t)$ with a limited spectrum of width $B$ is representable in the form of a series

$$f(t) = \sum_{n=-\infty}^{\infty} f\left(\frac{n}{2B}\right) \operatorname{sinc}(2\pi Bt - n\pi)\,, \qquad \operatorname{sinc}(x) = \frac{\sin x}{x}\,.$$

The theorem states in essence that any function is completely representable by the collection of its values selected at discrete points in time $t_n = n/2B$. If ultrashort pulses are emitted with amplitudes equal to the values of the function at the above discrete points in time, a receiver possessing a low-pass filter of spectral width $B$ will generate oscillations of the form $\operatorname{sinc}(x)$ and the sum of these oscillations will once again exhibit the undistorted function $f(t)$. This procedure of signal transmission and reception is explained in Fig. 1. Since the bandwidth of the low-pass receiver filter should not be smaller than the spectral width of the signal, attempts to narrow this band for an undistorted signal transmission are similar to endeavors to make a *perpetuum mobile* as warned by Kotel'nikov [1] in the formulation of the problem.

Interestingly, in 1936 Kotel'nikov tried to publish his theorem in the journal *Elektrichestvo* (Electricity). However, he was denied publication because the journal was overloaded with papers and his paper was only of narrow interest. If only those who turned him down knew what they were saying! What actually happens is that the theorem has more profound importance than the problem that led to its proof. In essence, it pointed the way for the representation of continuous functions in digital form and thereby came to be one of the theoretical foundations of numerical technology which has been rapidly advancing during the last decades. In the formulation of the problem of the digital representation of continuous functions, first of all there arises the question of how frequently the values of a function should be sampled to adequately represent its form. The first and naive answer is: the more frequently, the better. This signifies that the undistorted transmission of any message calls for rather frequent sampling. However, in communication systems we deal with signals of limited spectral width. Such signals cannot exhibit arbitrarily rapid variations in time. That is why the signal samples taken within too short a time interval may turn out to be little different from one another, and the use of their total collection is unnecessary. A function with a limited spectrum may significantly vary only within time intervals not shorter that the reciprocal of its spectral bandwidth. This was recognized by H Nyquist who was presumably one of the first to express the idea that the samples of a signal should be differed by time intervals equal to approximately the reciprocal of its spectral bandwidth [2]. Not infrequently this gives grounds, especially for Western scientists, to use the term 'Nyquist sampling rule'. However, Nyquist applied his reasoning to the problem of undistorted transmission of a telegraph (digital) signal. This problem is different from the problem of the undistorted transmission of an analogue signal, although they have much in common as pointed out by Professor D Lüke in his paper concerning the origin of the sampling theorem [3]. He noted that "V A Kotel'nikov was presumably the first scientist who rigorously formulated the sampling theorem and applied it to the theory and technology of communications". This statement gave grounds to award the Eduard Rhein Foundation prize 1999 for basic research to Kotel'nikov.

A similar theorem had been known to mathematicians. In particular, in 1915 E Whittaker proved it when investigating the approximation problem for entire functions of finite

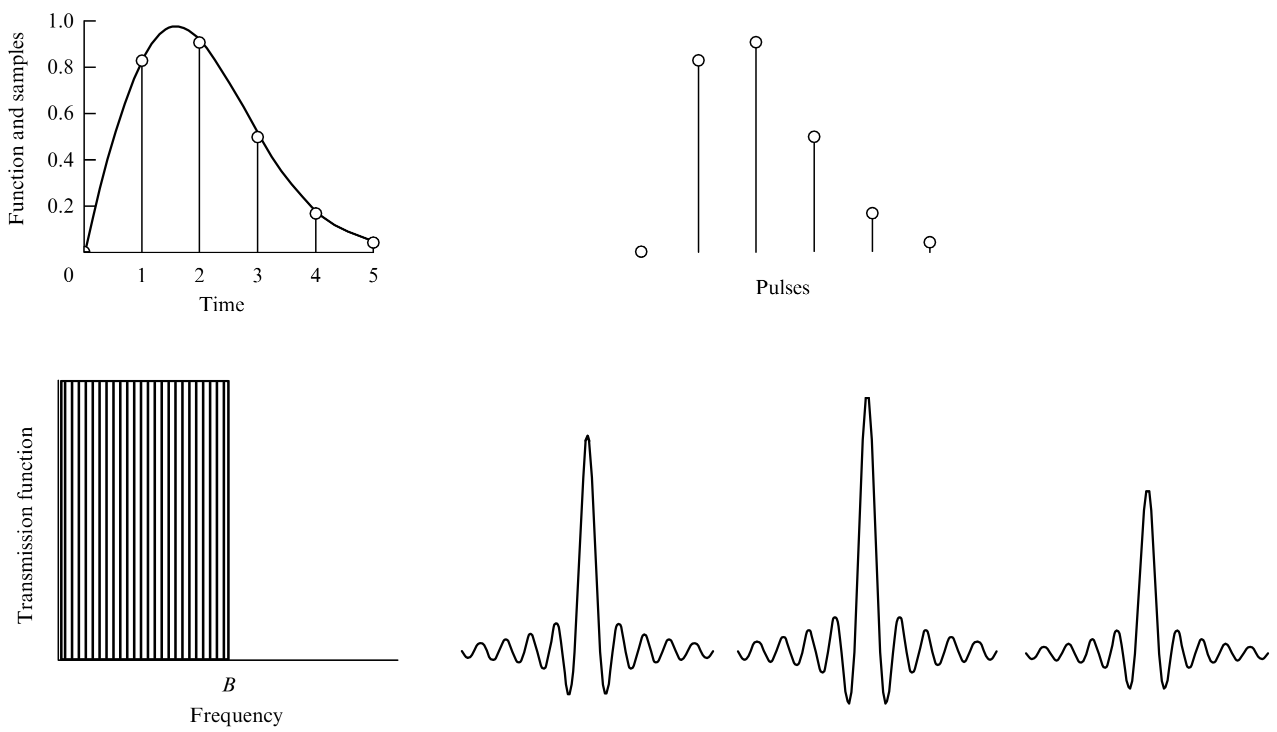


**Figure 1.** Schematic diagram highlighting the sampling theorem.

degree [4]. Kotel'nikov was not familiar with that work. However, in mathematics this is just one of many ordinary theorems. In communication theory and digital technology this theorem is central, and the credit for its proof undoubtedly goes to Kotel'nikov. Unfortunately, the problems with the publication of his theorem prevented the members of the broad scientific community from familiarizing themselves with it. It was not until C Shannon proved the sampling theorem anew in 1948 [5] that it became widely known. At present, this theorem is frequently referred to as the Whittaker – Kotelnikov – Shannon sampling theorem [6].

The Kotel'nikov theorem can be extended to any functions possessing limitations in some space [7]. There is an adjoint theorem which pertains to functions limited in time [8]. In particular, it is possible to produce short pulses by generating oscillations at discrete frequencies. The antenna directivity pattern is the Fourier transform of the currents whose spatial distribution is bounded by the antenna aperture. On these grounds, the directivity pattern can also be represented as a discrete series [9]. In the processing of images a demand arises for digitizing them, and in this case Kotel'nikov's theorem is one of the most important instruments for effecting this operation.

An interesting example is provided by a somewhat unexpected application of the theorem to the description of signal dispersion [10]. As is well known, this effect occurs in wave propagation through media where the phase velocity is frequency-dependent and manifests itself in that the shape of signals is distorted in their propagation. Figure 2a shows an undistorted signal produced by linear frequency modulation in a band $B$. It possesses the shape of sinc $(\xi)$ and is therefore represented by one Kotel'nikov component. During propagation in plasma, the signal shape is distorted to assume the form plotted in Fig. 2b. This distorted signal now possesses many Kotel'nikov components, their number and amplitudes reflective of the degree of signal distortion. From their parameters it is possible to recover the signal shape [10].

### 3. Theory of potential noise immunity

In this section we dwell on the next classic Kotel'nikov work concerned with the limiting sensitivity of receiving systems. At the end of the 1930s there arose a crisis in the improvement of noiseproof feature of communication systems. Technical contrivances of all kinds ran across some limit which hindered further increases in receiver sensitivity. This brought up the natural question: did it result from the insufficient resourcefulness of engineers or did there exist some basic reasons that impose a limit on the noise immunity of the systems involved? The answer to this question was provided in Kotel'nikov's doctoral dissertation "The theory of potential noise immunity" written in 1946 and successfully defended in 1947. The aim of the work was "to elucidate whether it is possible to lessen the influence of interference by improving receivers for the existing types of signals. Can noise abatement benefit from a change in the signal form? What signal forms are optimal for the purpose?" [11].

Much in the work under discussion was radically new and unusual to the practising engineers of that time. First of all, there was the introduction of orthonormal time functions $C_k(t)$ in terms of which the signal could be expanded. The signal $A_j(t)$ is represented as a sum:

$$A_j(t) = \sum a_{jk}\, C_k(t)\,.$$

Different signals differ by the $a_{jk}$-coefficient set. In the case of a limited number of basis functions $C_k(t)$, this expansion would be referred to as the signal representation in the finite-dimensional Euclidean space (Hilbert space) [12]. The signals may be treated as the vectors in this space. An example of such geometric representation is given in Fig. 3. It is pertinent to

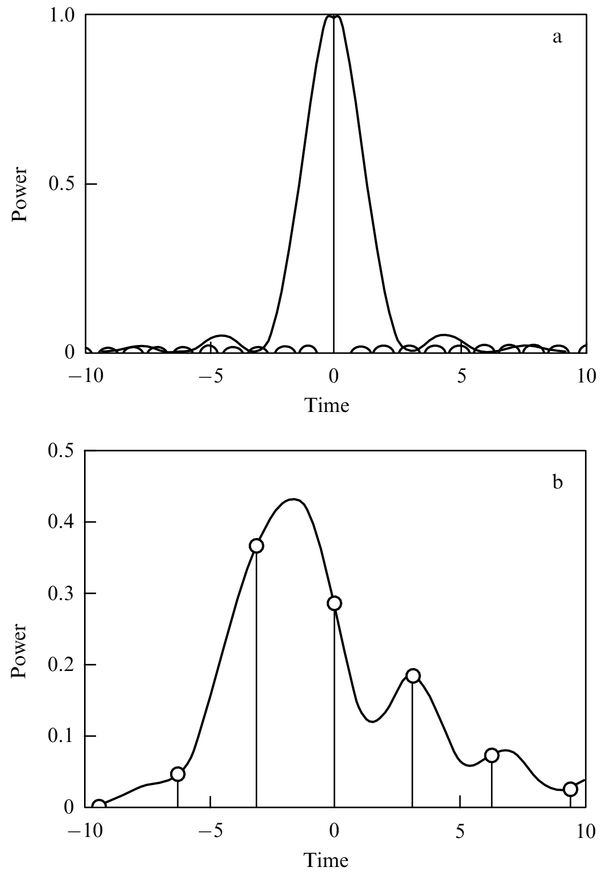


**Figure 2.** Sampling theorem and signal dispersion.

note that the illustrations like those in Fig. 3 are absent from the thesis although its author quite frequently addresses himself to the geometric representation of the signal. In practical calculations, Kotel'nikov took advantage of Fourier series, which is a natural tribute to the conventional spectral representation of the signal.

The first problem considered in the work was that of signal identification. Its essence is represented in simplified form in Fig. 3. A mixture of signal and noise $\mathbf{X}$ arrives at the receiver input. What should be the response at the receiver output — is this signal $\mathbf{A}_1$ or $\mathbf{A}_2$? Clearly, the answer will be $\mathbf{A}_1$

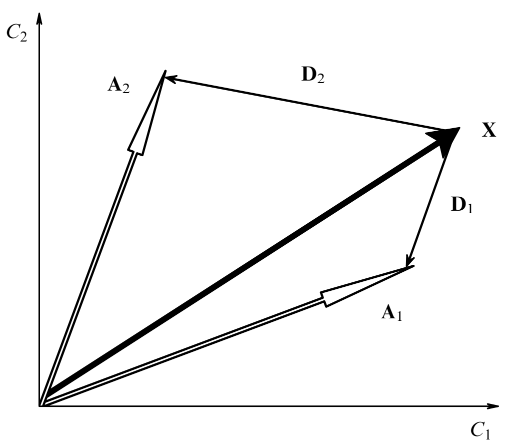


**Figure 3.** Geometric representation of signals.

if the inequality $|\mathbf{D}_1| < |\mathbf{D}_2|$ is valid for Euclidean distances. However, the validity or invalidity of this inequality is statistical in nature, because some possibility exists that the inequality under discussion is not fulfilled due to random noise behavior. It is therefore reasonable to expect that correct signal extraction against the noise background is probabilistic in nature. Hence follows the concept of an ideal receiver as yielding the minimal number of incorrectly reproduced messages upon noise induction. Potential noise immunity is characterized by the least possible distortions. It is equal to the probability of incorrect reproduction and in the case of Gaussian noise with a uniform spectrum is defined by the ratio between the specific energy and the noise intensity $\sigma^2$:

$$\alpha = \frac{1}{2\sigma^2} \int_{-T/2}^{T/2} \left| \mathbf{A}_1(t) - \mathbf{A}_2(t) \right|^2 \mathrm{d}t \, .$$

Here, $T$ is the signal duration. This relation takes on quite a simple form in the typical case for radar, when $\mathbf{A}_2(t) = 0$:

$$\alpha = \frac{Q}{2\sigma^2} \, ,$$

where $Q$ is the energy of the signal. In this case, ''the potential noise immunity is defined only by the energy of the signal and is absolutely independent of its form'' [11]. Modern radar experts would say that the parameter $\alpha$ is the signal-to-noise ratio which defines the probability of false alarm. For a high signal-to-noise ratio, the probability of correct signal extraction is close to unity, and the probability of false alarm tends to zero. More recently (1948), Shannon obtained the corresponding results for a broader class of noise. It is significant that the decisive role in potential noise immunity is played by the energy of a signal rather than its power. This circumstance is not universally recognized by everyone. Modern techniques of signal production are quite often reliant on its moderate power, while the signal extraction procedure itself involves compression (optimal filtration) in the receiving device [8]. The final answer to the question posed in the first pages of the Kotel'nikov's thesis reduces to the following statement: to improve the noiseproof feature of a communication system requires increasing the signal-to-noise ratio which turns out to be the crucial parameter defining the probability that the signal is correctly extracted against the noise background.

In the following parts of the work, the signal identification problem discussed in the foregoing was supplemented with the problems of parameter assessment and filtration. Thereby addressed were the main problems of statistical radio engineering. This underlies the statement about the fundamental character of Kotel'nikov's doctoral dissertation.

It is pertinent to note that mathematicians had obtained several basic results in the probability theory and the theory of random processes, which were of prime importance to the filtration theory, the parameter assessment theory, the theory of statistical solutions, etc., by the time Kotel'nikov wrote his work. Here, we are faced with a situation that is similar to the situation with the sampling theory. The results obtained by mathematicians did not find their way to the consumer, and the efforts of different specialists were called for to give them practical significance. In 1998, S Verdu published a paper dedicated to the fiftieth anniversary of Shannon's theory [13]. It said, in particular, that the greatest contribution to the introduction of the theory of random processes into the toolkit of communication engineers had been made by Wiener [14] and Rice [15]. However, Wiener's paper, published in 1949, could not have been familiar to Kotel'nikov in 1946. As regards Rice's work, it was published in 1944, and it was the only paper referred to in Kotel'nikov's thesis for doctorate. There were no other references, because there were no predecessors. That is why it is fair to say that Kotel'nikov should be regarded as one of the founders of statistical radiophysics and radio engineering. For some reason, this outstanding role of his is not broadly reflected in the scientific literature. The fact that the statistical views were not widespread among radio engineers is shown by the mode of Kotel'nikov's work presentation itself. Despite the fact that it constantly deals with random processes, the terms correlation, spectral density, etc. are not encountered, although they are implicitly present. The procedure of decision making itself is based on the Bayes strategy, but this is not mentioned in the text, and the formula for the *a priori* probability is derived simply from 'reasonable' considerations.

As already noted, *The Theory of Potential Noise Immunity* was published only in 1956, when the works of many other authors had gained wide recognition. That is why this work is well known to only those who are 'well informed' and in recent years has been sometimes cited primarily by Russian scientists. And this comes as no surprise. Science is advancing.

**4. Planetary radar**

Planetary radar is another outstanding achievement in Kotel'nikov's scientific work. In the 1960s, progress in rocket and space technology opened up the possibility of launching space vehicles to other planets of the Solar system. To control the flight of such a spacecraft required a sufficiently thorough knowledge of planet locations. The astronomical observations performed by that time provided reliable data about the relative dimensions of the Solar system. However, successful interplanetary navigation called for a good knowledge of the absolute dimensions of the system. The main scale quantity characterizing the dimensions of the Solar system is the astronomical unit (AU), which is equal to the major semiaxis of the elliptic orbit of the Earth, or the average distance from the Earth to the Sun (about 150 million km). It can be calculated if the distance between two planets is known. Radar exactly furnishes this possibility. The team supervised by Kotel'nikov took on the task of its implementation. In this case, Kotel'nikov proved to be a remarkable organizer, and not only an outstanding scientist. At that time, a long-range space communication center was being built in Evpatoriya (the Crimea), which consisted of a high-power transmitter ($\approx 10$ kW) at a wavelength of 39 cm, as well as the large transmitting and receiving antennas ADU-1000 with an effective area of about 1000 m$^2$. To successfully implement the radar required devising a wide range of equipment for filtering signals and measuring their spectrum, frequency, etc. It is worth mentioning that there were no computers at the beginning, and many algorithms could not be realized by software-based techniques. The only way was to make the corresponding facilities on one's own.

That planetary radar was an intricate matter at that time was evidenced by foreign experience. The first attempts of Venus's radar location were undertaken in the USA (1958) [16] and England (1959) [17]. However, the results of these experiments turned out to be erroneous. This was also confirmed by the first experiment of Kotel'nikov's group. Early in the work the capabilities of the radar set were so low that extracting the signal required accumulating it for several hours. However, as the technology developed (which involved increasing the transmitter power, equipping the receiver with low-noise paramagnetic amplifiers, introducing linear frequency modulation, improving the methods of signal extraction, etc.), they succeeded in realizing a relative measurement accuracy of about $10^{-8}$ for interplanetary distances. This afforded determination of the astronomical unit with an accuracy on the order of 1 km. This accuracy is thousands of times higher than that attained by astronomical methods. At the XVIth General Assembly of the International Astronomical Union (1967) it was accepted that 1 AU $= 149{,}597{,}870 \pm 2$ km for a speed of light $c = 299{,}792{,}558 \pm 1.2$ m s$^{-1}$. The time of radio wave propagation was determined so accurately that the accuracy with which the speed of light was known turned out to be significant in the time – distance translation. It is noteworthy that planetary radar investigations were simultaneously pursued in the USSR and the USA. Competition existed between scientific teams and, naturally, many results were similar.

Apart from Venus, radar measurements were made of Mars, Mercury, and Jupiter. These measurement data were also used to make more precise the astronomical unit. The high precision of the radar measurements permitted constructing the theory of planetary motion, which was more exact than the theory relying on optical data. To construct this theory, account had to be taken of the effects of the general relativity theory. Planetary radar thereby came to be one of the means of verifying the implications of general relativity.

Radar observations also enabled revising other parameters of the planets. This is especially true for Venus whose radius, period, and sense of rotation were refined. In particular, the rotation of Venus was found to be of opposite sense (relative to the sense of its orbital motion around the Sun) and its period was measured at 243.04 days. Interestingly, this value is quite close to the synodic resonance with a rotation period of 243.16 days, whereby the same side of Venus would be facing the Earth at every inferior conjunction. The results of planetary radar are described in greater detail, for instance, in Ref. [18].

**5. Radar cartography of Venus**

In the implementation of this work, which brought fame to the Soviet space program, V A Kotel'nikov was not its formal supervisor. However, the program of radar-assisted cartography of Venus could hardly have been realized without his active participation. In doing this there was a need to 'synchronize', apart from the Institute of Radioengineering and Electronics of the USSR Academy of Sciences, the work of the S A Lavochkin Research and Production Association, the Special Design Bureau of the Moscow Power Engineering Institute, and several other industrial and academic organizations. This was within the power of only such a prominent and authoritative personality as Kotel'nikov.

In 1983, the artificial satellites 'Venera-15' and 'Venera-16', each having aboard a radar with a synthetic aperture and an altimeter, were sent into Venus orbit. The radar enabled obtaining an image of the planet's surface, permanently screened by clouds and therefore invisible in the optical range. In this way, they succeeded in mapping the planet surface with a spatial resolution of 1 – 2 km, and the altimeter provided data about the relief with a resolution of 230 m in altitude. And so for the first time humankind learned about the surface structure of the northern part of Venus over an area of 115 million km$^2$ (25% of the Venus total surface). Unquestionably, this was an outstanding achievement. Several years later this research was continued in the USA during the Magellan mission, when they managed to obtain images of almost the entire surface of Venus with a spatial resolution of about 100 m. This mission was planned with the inclusion of the results of the Soviet space program. The results of the Venera-15 and Venera-16 missions are described in greater detail in Ref. [19].

We emphasize that, apart from the radar survey itself, the radar data processing and the construction of planet images were also a problem. At that time, one of the most complicated operations was the Fourier analysis. The computers which were at the disposal of researchers were too weak to perform this operation in a reasonable time. It was not without reason that optical processors were still employed at that time to process the data from radars with synthetic aperture, in which the Fourier transform of a radar hologram was effected with a lens. The project participants devised and made a special-purpose Fourier processor which permitted increasing the SM-4 computer speed up to $5 \times 10^7$ operations per second by this algorithm and thereby performing the all-digital radar data processing and surface image construction. That was the USSR's first experience in the digital processing of radar image, which was subsequently taken into account in the production of the Almaz radar data processing programs.

### 6. Conclusion

In so brief a report it is hard to outline all the results of the activities of a personality like V A Kotel'nikov in scale and depth of thought. We have not, in particular, touched upon his basic works in the area of cryptography, his role in the design and implementation of the Moscow – Khabarovsk radio communication system in the pre-war years, his contribution to the theory of parametric amplifiers, his role in the making of the systems of communication with deeply submerged submarines, and many other things. It is pertinent to note that Kotel'nikov initiated time and again new lines of research performed both at the Institute of Radioengineering and Electronics of the Russian Academy of Sciences, which he directed for many years, and in other organizations. Special mention should be made of his role in the development of space research, which he played as Vice-President of the USSR Academy of Sciences and Chairman of the Interkosmos Council.

To summarize, it is valid to say that Vladimir Aleksandrovich Kotel'nikov was an outstanding scientist and engineer of the 20th century — one of the founders of digital signal processing technology, information theory, statistical radiophysics, radio engineering, and radar astronomy. This brief list alone makes it clear that we were dealing with a prominent personality in the history of our country and science, who made an enormous contribution to the progress of science.

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