

EXPLANATORY NOTE

The study of probability theory and its applications has had a long and illustrious history in Russia, beginning in the earliest days of the 18th century, and continuing in an unbroken line down to the present generation. In our time we have seen a realization that in many aspects of science, technology, and human behavior the element of randomness is so fundamental that often one can hardly define a meaningful problem, much less solve it, without using probability theory. During the rapid technological developments of the World War II period, the communication and detection arts underwent such a realization, and as would be expected, statistical communication theory (or information theory) has occupied some of the best minds among mathematicians and engineers in the Soviet Union just as it has elsewhere.

One of the most important Soviet contributions, and one that was until recently virtually unknown outside the U.S.S.R., was the 1947 doctoral dissertation of V. A. Kotel'nikov, at that time a 40-year-old communications engineer, who had already in his younger days (1933) become well known for his work on sampling theorems for band-limited functions. Kotel'nikov's dissertation constituted an extensive analysis of the effects of additive gaussian noise on communication systems, and of what could be done at the receiver to minimize them. Unlike Shannon's information theory, he did not go extensively into the implications of a freedom to choose complicated transmitter signals.

Many Soviet contributions to the statistical communication art are fairly well known to us. Every student of these matters knows the names Khinchin and Kolmogorov as partners with Western mathematicians (notably Norbert Wiener) in the early development of spectral and filtering theories for random functions. Yet few of us have been aware that there existed in 1947 in this dissertation a statistical analysis of communication problems using what we now call decision theory techniques and anticipating by several years much of the work of Woodward, Davies, Siegert and others, with which we are more conversant.

This book is a verbatim translation of "The Theory of Potential Noise Immunity", published by the State Power Engineering Press in 1956. As the Author's Preface has just indicated, it is essentially identical to the 1947 dissertation. In preparing this English edition, no technical editing has been done other than the correction of misprints. The present volume thus retains the exact flavor of the original, allowing one to see from hindsight which of Kotel'nikov's many highly original ideas have been developed further and which have not.

By no means all of Kotel'nikov's results have since been obtained independently by others, and thus the volume should be of much more than just historical interest.

Perhaps the reader will be aided by the following few comments which should make the unfamiliar terminology a little easier to follow, and should clarify the relationship between this and other works in the communication theory field.

First of all there is the question of just what is meant by "noise immunity". As used here, it is a generic term with *a*. different meaning for different situations. For Part II, the case in which communication takes place by transmitting one out of a finite number of possible signals the term refers to probability of no error. (Part II discusses what we would now call the "multiple-alternative decision" problem.) Part III treats the situation in which a continuum of transmitted signals is assumed (a parameter λ ranging over some interval taking the place of the previous discrete index of the possible signals); i. e. , the problem of "parameter estimation". Here a greater noise immunity refers to a decrease in mean square value of the error between the value of λ indicated by the receiver and that actually transmitted. And then when the author treats in Part IV the case of signalling using waveforms (the parameter now being replaced by a function of time in some time interval) an increased noise immunity refers to a decrease in the average noise power that additively corrupts the receiver output. In other words, the author is dealing with the mean-square error between the modulating signal entering the transmitter, and that reproduced by the receiver. Note that the author never says that noise immunity "is" one of these three things, but rather "is characterized by" one of them. This usage persists in the large number of Soviet papers that have continued Kotel'nikov's work.

The development presented here is notable in its absence of any dependence on an advanced mathematical background. The reader possessing a passing familiarity with Fourier series, discrete and continuous probabilities and probability densities (simple, joint, and conditional) and the notion of statistical independence will have no trouble. At several points some known results of probability theory are invoked without reference or proof. (One that the beginning reader might not be familiar with is the Central Limit Theorem, Equation 2-33.) However, these instances are rare; by and large the treatment is completely self-sufficient.

Kotel'nikov made extensive use of geometric models of the signalling and detection processes as operations on vectors in multi-dimensional space, an artifice that Shannon introduced later. The reader will find these geometric interpretations very helpful. The material of each chapter is reviewed in terms of the geometric model at the end of the chapter.

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AUTHOR'S PREFACE

This book is the author's doctoral dissertation, presented in January, 1947, before the academic council of the Molotov Energy Institute in Moscow. Despite the fact that many works devoted to noise immunity have appeared in the time that has elapsed since the writing of this dissertation, not all of the topics considered in it have as yet appeared in print. Considering the great interest shown in these matters, and also the number of references made to this work in the literature, the author has deemed it appropriate to publish it, without introducing any supplementary material. However, in preparing the manuscript for publication, it was somewhat condensed, at the expense of material of secondary interest. Moreover, Chapter 2, which contains auxiliary mathematical material, has been revised somewhat, to make it easier reading, and some of the material has been relegated to the appendices.

- The author

THE THEORY OF OPTIMUM NOISE IMMUNITY

In 1947 V. A. Kotel'nikov, a communications engineer already well known for his work on sampling theorems for band-limited functions, published a doctoral dissertation that has proved to be one of the most important Soviet contributions to the statistical communications art. Because this work was virtually unknown outside the U. S. S. R. until quite recently, few Western scientists were aware that Dr. Kotel'nikov had developed a statistical analysis of communication problems (using what we now call decision theory techniques), which anticipated by several years much of the work by Western communication experts.

This book is a verbatim translation of THE THEORY OF POTENTIAL NOISE IMMUNITY, published in Moscow in 1956, and is essentially identical with the 1947 dissertation. Kotel'nikov's paper extensively analyzes the effects of additive gaussian noise on communication systems and determines what can be done at the receiving end to minimize them.

The author's approach is always that of a practical engineer. He invokes only the most elementary notions of probability theory; the steps in his analysis are easy to follow; and his primary intention is to establish the behavior of communications systems at a level that is of practical use to the engineer. The development is also notable for its lack of dependence on advanced mathematics. The reader who is passingly familiar with Fourier series, discrete and continuous probabilities and probability densities (simple, joint, and conditional), and the notion of statistical independence will have no trouble with the material.

Dr. Kotel'nikov made extensive use of geometric models of the signalling and detection process as operations on vectors in multi-dimensional space, an artifice that Shannon introduced later. The reader will find these geometric interpretations very helpful. The subject matter of almost every chapter is reviewed in terms of the geometric model at the end of the chapter.

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When Dr. Kotel'nikov's paper first received limited circulation in Russia, the approach was startlingly new. Since that time, most of the major concepts in his work have been obtained independently in the West, although many of his results have yet to be worked out. Much of the material has appeared in the professional literature, but has not previously been published in book form.

Both as an historical document and as a reference work, THE THEORY OF OPTIMUM NOISE IMMUNITY should prove extremely helpful to students and research workers involved in communication theory or the mathematical analysis of communication systems.



V. A. KOTEL'NIKOV

(Photograph courtesy of ELECTRONIC DAILY, a Hayden Publishing Company Publication)

Vladimir Aleksandrovich KoteFnikov was born in 1908 in Kazan' and received his education in electrical engineering at the Molotov Power Engineering Institute in Moscow. He has been on the research and teaching staff of that institution since his graduation in 1931, and since 1947 has headed the Chair of Radio Engineering.

He has received two Stalin prizes and in 1953 was elected Academician, the highest rank in the U. S. S. R. Academy of Sciences. He is one of the three or four Soviet electronics engineers ever to have received this honor. He is a member of the Presidium of the Popov Society, the radio engineering and electronics professional society of the Soviet Union. Since 1954 Academician Kotel'nikov has been Director of the Institute of Radio Engineering and Electronics, a large research center of the Academy of Sciences.

The **THEORY** of **OPTIMUM** **NOISE** **IMMUNITY**

V. A. KOTEL'NIKOV

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